ZCAS University

FINAL EXAMINATION

CAS2282 STATISTICS

29ᵀᴴ MAY 2024

TIME 08:30-11:30 HRS

TIME ALLOWED: THREE HOURS (plus 5 minutes to read through the paper)

INSTRUCTIONS:

1. Section A: this question is **compulsory** and must be attempted.

2. Sections B: Answer Three (3) questions from this section.

3. This examination paper carries a total of **100 marks**.

4. Candidates must **not turn this page** until the invigilator tells them to do so.

Question 1

a. On each of four days next week (Monday through Thursday), Earl will shoot six free throws. The probability of success is p=0.37. Compute the probability that on any day Earl obtains exactly two successes. **(10marks)**

b. A corporation regularly takes deliveries of a particular sensitive part from three subcontractors. It found that the proportion of parts that are good or defective from the total received were as shown in the following table:

	subcontractor		
	A	B	C
part good	0.27	0.3	0.33
defective	0.02	0.05	0.03

I. If a part is chosen randomly from all those received, what is the probability that it is defective? **(10marks)**

II. If a part is chosen randomly from all those received, what is the probability it is from subcontractor B? **(10marks)**

III. What is the probability that a part from subcontractor B is defective?**(10marks)**

(Total: 40 marks)

SECTION B: Attempt any THREE questions in this section

Question 2

A. A manufacturer knows that the numbers of items produced per hour by machine A and by machine B are normally distributed with a standard deviation of 8.4 items for machine A and a standard deviation of 11.3 items for machine B. The mean hourly amount produced by machine A for a random sample of 46 hours was 130 units; the mean hourly amount produced by machine B for a random sample of 40 hours was 120 units.

Required: test the hypothesis that there is no difference between the means of the two machines, use the 97% level of significance. Assume equal variance. **(10marks)**

B. Based on an examination of past records of a corporation's account balances, an auditor finds that 15% have contained errors. Of those balances in error, 60% were regarded as unusual values based on historical figures. Of all the account balances, 20% were unusual values. If the figure for a particular balance appears unusual on this basis, what is the probability that it is in error? **(10marks)**

(20 marks)

Question 3

A. ZCAS university offers admissions for an MBA program has determined that historically applicants have undergraduate grade point averages that are normally distributed with standard deviation 0.45. From a random sample of 18 applications from the current year, the sample mean grade point average is 2.90.

Required: Find the confidence interval for the population mean at the 5% significance level. **(10marks)**

B. Andrew Whittaker, computer center manager, reports that his computer system experienced three component failures during the past 100 days.

i. What is the probability of no failures in a given day? **(5marks)**

ii. What is the probability of one or more component failures in a given day? **(5marks)**

(20 marks)

Question 4

A. For a random sample of 120 British entrepreneurs, the mean number of job changes was 1.91 and the sample standard deviation was 1.32. For an independent random sample of 80 British corporate managers, the mean number of job changes was 0.21 and the sample standard deviation was 0.53. population variance unknown and not equal

Required: Test, at the 3% level, Test the null hypothesis that the population means are equal against the alternative that the mean number of job changes is higher for British entrepreneurs than for British corporate managers.

(10marks)

B. Customers arrive at a photocopying machine at an average rate of 2 every five minutes. Assume that these arrivals are independent, with a constant arrival rate, and that this problem follows a Poisson model, with X denoting the number of arriving customers in a 5-minute period and mean $1/2$. Find the probability that more than two customers arrive in a 5-minute period.

(10marks)

(20 marks)

Question 5

A fast-food chain decided to carry out an experiment to assess the influence of advertising expenditure on sales. Different relative changes in advertising expenditure, compared to the previous year, were made in eight regions of the country, and resulting changes in sales levels were observed. The accompanying table shows the results.

Increase in Sales% (Y)	2.4	7.2	10.3	9.1	10.2	4.1	7.6	3.5
Increase in advertising expenditure% (x)	0	4	14	10	9	8	6	1

a. Estimate by least squares the coefficients of the fitted model. **(10marks)**

b. Find and interpret the coefficient of determination. **(10marks)**

(20marks)

END OF EXAMINATION

Formulae list

$$\text{var}(\hat{p}_x - \hat{p}_y) = \frac{\hat{p}_x(1-\hat{p}_x)}{n_x} - \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}$$

$$v = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\left(\frac{s_x^2}{n_x}\right)^2 \frac{1}{(n_x-1)} + \left(\frac{s_y^2}{n_y}\right)^2 \frac{1}{(n_y-1)}} \quad \text{or} \quad v = \left(1 + \frac{2}{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}\right)(n_x - 1)$$

$$\text{var}(\bar{x} - \bar{y}) = \frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}$$

$$P(B/A) = \frac{P(A/B)P(B)}{P(A)}$$

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0,1,2,\ldots,n$$

$$P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$$

$$P(x) = \frac{\binom{r}{x}\binom{N-r}{r-x}}{\binom{N}{n}}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

$$\chi^2_{(n-1)} = \frac{(n-1)s^2}{\sigma^2} = \frac{\sum(x_i - \bar{x})^2}{\sigma^2}$$

$$f(x) = \lambda e^{-\lambda x}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

$$S_{b_1} = \sqrt{\frac{\sigma^2}{\sum(x-\bar{x})^2}}$$

$$F = \frac{ESS/k-1}{RSS/n-k}$$

$$\sigma = \sqrt{\frac{\hat{p}_0(1-\hat{p}_0)}{n_x} + \frac{\hat{p}_0(1-\hat{p}_0)}{n_y}}$$

$$\hat{p}_0 = \frac{n_x\hat{p}_x + n_y\hat{p}_y}{n_x + n_y}$$

$$s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x + n_y - 1}$$